

In Search For Preece and Onderdonk

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The authors began to collaborate on the temperature effects of current on PCB traces in late 2014. Among the topics they explored is the fusing current a PCB trace can carry. The early work on this topic goes back to W. H. Preece in the 1880's, and I. M. Onderdonk, possibly in the 1920's. Preece's work has been well documented, although until recently copies of that documentation have been difficult to obtain. There is growing speculation, however, that Onderdonk may never have published his source data under his own name.

In this paper the authors (a) discuss the background of Preece, his motivating factors, and links to his source data; (b) discuss what can be inferred about Onderdonk, his likely motivating factors, and links to early work based on his equation; (c) outline the critical assumptions behind these formulas, and (d) in the absence of any Onderdonk source data, independently derive Onderdonk's equation.

W. H. Preece:

In the 1880's, Sir William Henry Preece was a consulting engineer for the British General Post Office, and became Engineer-In-Chief in 1892 [1]. At that time, the Post Office was responsible for the telegraph (and later wireless telegraph) system in England. He published three papers [2] in the Proceedings of the Royal Society of London in the 1880's that formed the basis for his famous equation:

$$I = a * d^{3/2} \quad [\text{Eq. 1}]$$

where d is the diameter of the wire in inches, a is a constant (10244 for copper), and I is the fusing current in Amps. A little algebra transforms this equation to:

$$I = 12277 * A^{3/4} \quad [\text{Eq. 2}]$$

where A is the cross-sectional area in inches².

Interestingly, in the 1890's Preece became an ardent supporter, both politically and financially, of Guglielmo Marconi and his work on trans-oceanic wireless telegraphy.

Preece was motivated by the problem of lightning striking telegraph wires. The early form of lightning arrester used in that period is shown in Figure 1, reproduced from Preece's 1883 paper. Most of the energy from the strike was intended to flow through the capacitor, but some was left to travel onto the "cable," which led to the telegraphy equipment. Preece wanted to determine the best material, and its size, for the lead from the arrester to the cable. The wire should be able to carry the normal current for telegraph operations, but fuse at currents slightly above that level.

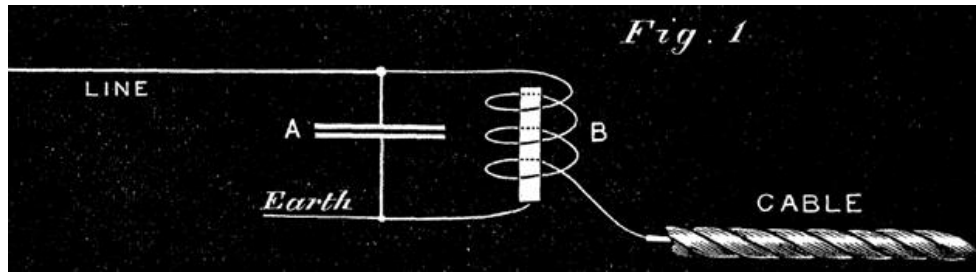


Figure 1

Early form of lightning arrester. Source: Preece's 1883 paper.

Preece did not approach this problem analytically. He was, what we might call today, a “lab rat” and did all his experiments in the lab. He would apply current to a test wire until the wire began to glow. He would call that point the “fusing” point (more appropriately the “melting” point). He looked at a large selection of materials and provided the constant (a in Equation 1) for each material. He concluded that “the best metal to use for small diameters was platinum, and for large wires tin” [1887 paper]. The final constants he derived were summarized in his 1888 paper, which is where the value 10244 is found.

I. M. Onderdonk:

Onderdonk, however, is much more obscure. Speculation is growing that Onderdonk never published his work under his own name, and we have uncovered no citations for any work by Onderdonk himself. His equation is offered almost as a given, in much the same way we reference Ohm's Law today. His equation appears in several papers and sources [3 through 7], usually in the form

$$33 \left(\frac{I}{A} \right)^2 S = \log_{10} \left(\frac{t}{274} + 1 \right) \quad [\text{Eq. 3}]$$

for $T = 40^\circ \text{C}$, but occasionally in the more general form:

$$33 \left(\frac{I}{A} \right)^2 S = \log_{10} \left(\frac{t}{234 + Ta} + 1 \right) \quad [\text{Eq. 3a}]$$

Where: I = the current in Amps

A = the cross-sectional area in circular mils (note 1)

S = the time in seconds the current is applied

t = the rise in temperature from the ambient or initial state (see note 2)

Ta = the reference temperature in degrees C

Some publications erroneously cite Onderdonk's name in a citation. We will repeat two of them here, anonymously to protect the authors:

Onderdonk, J. M., Short-Time Current Required to Melt Copper Conductors, Electrical World 121:26, 98 (24 June 1944).

I. M. Onderdonk and E. R. Stauffacher, "Short Time Current Carrying Capacity of Copper Wires", G.E.Review, pp.326 1928

The first [4] is an anonymous (not by Onderdonk, as the citation suggests) single page nomograph inserted in the magazine. It is provided here in Appendix 1. There are notes on the face of the nomograph that say (somewhat cryptically, and among other things) "...based on an equation by I. M. Onderdonk," and "Chart from engineering department of a midwestern utility."

The second [5] is a respectable article written by E. R. Stauffacher (but NOT co-authored by Onderdonk). It contains a different nomograph (reproduced here in Appendix 2). Stauffacher comments that the "Values of short-time current required to melt copper conductors, as given by the chart shown in Fig. 1, were calculated from the formula..." and then he offers Onderdonk's equation. A footnote to the equation says "This formula was developed by I. M. Onderdonk."

It is tempting to speculate that, since Stauffacher is the earliest reference we have found, Onderdonk may have been a contemporary of Stauffacher. Stauffacher, himself, was employed as Superintendent of Protection at Southern California Edison when he wrote his article (in 1928). Perhaps Onderdonk was an engineer at SCE at the same time.

Stauffacher's article offers a clue as to what the motivation was for Onderdonk's equation. The concern was that under certain conditions there may be a short circuit that occurs between a high-voltage transmission line and the copper wires attached to the insulators or poles supporting the line. These copper wires needed to carry that short-circuit current for the short period of time "required to clear the faulty line from the system." Thus, the desirability of having a variable for time (S in the equation).

Cautions:

It is important to understand what constitutes the "melting" of a copper wire. When we apply heat to a wire (or any solid for that matter), there are two stages we can consider. The first is that the wire heats up from the initial temperature to the melting temperature. Let's call this time t_1 . Then additional heat (and time, say time t_2) is required to change the wire from a solid to a liquid (i.e. to actually melt the wire.) This second amount of heat is referred to as the heat of fusion (Note 3.) Time t_1 is required to heat the wire TO the melting temperature, and time $t_1 + t_2$ is required to actually melt the wire from the initial temperature.

Preece clearly refers to time t_1 . He raised the current to the point where the wire glowed. Onderdonk's equation also only applies to time t_1 . This is inferred from most (but not all) of the references and is confirmed by the derivation in the next section.

Unfortunately the semantics found in some of the articles seems to confuse this point. For example, the nomograph in Electrical World is titled "Short-Time Current Required to **Melt** Copper Conductors" (emphasis added.) Stauffacher refers to the "short-time current required to melt copper conductors." But since both these are based on Onderdonk's equation, they actually only refer to the time to bring the temperature TO the melting point.

Preece's equation is based on slowly bringing the current up to the melting point. But he did not bring the variable of time into the equation. There would have been some cooling effects that occurred while

he was heating the various materials, but this was of little consequence in what he was trying to discover.

Onderdonk, on the other hand, specifically brought time into the relationship. Thus he had to make some assumption about the cooling effects that might have coexisted with the heating effects. His assumption was that there were no cooling effects (from conduction, convection, or radiation.) Thus, the heating was rapid (as might happen in a short-circuit situation.) That is why the references say that Onderdonk's equation is only valid for short periods of time, typically less than 10 seconds. People can quibble about this time. For example, Adam [8] suggests that cooling effects can begin to affect the results in as short as one or two seconds of time. In any event, Onderdonk's equation becomes less and less reliable as time increases.

Derivation of Onderdonk's Equation:

We are aware of only one serious attempt at independently deriving Onderdonk's Equation (there might well have been other attempts we are unaware of.) Babrauskas and Wichman [5] did so with interesting results. Their equation 6 derived the time to heat a copper wire to the melting temperature. Their equation 7 derived the time to melt the wire. Their equation 8 was the sum, which they then compared to Onderdonk's equation and remarked that their result was 17% higher than Onderdonk's. We believe that that comparison is not appropriate and they should have compared their equation 6 with Onderdonk's. Our comparison of their equation 6 with Onderdonk's suggests that their equation is just 4% below Onderdonk's, acceptably close.

Here is a list of variables used:

I	(DC) current	Ampere	
t	time	sec	
T	wire temperature (avg.)	deg C	
Θ	Temperature rise $T - T_{ref}$	Degree C	
L	Length of wire	meter	
M	Mass of wire	kg	
R	Electric resistance of the wire	Ohm	
c_p	Specific heat capacity (copper)	J/(kg*K)	385
ρ	density of wire material (copper)	kg/m ³	8900
ρ_{20}	electric resistivity @20 °C (copper)	Ohm*m ² /m	$1.72 \cdot 10^{-8}$
α	Temperature coefficient of resistivity	1/°C	.00393@20°
A	Cross-sectional Area	m ²	
K	Temporary utility variable	-	

Basic equation:

The following equation (see Note 4) describes an equality in a piece of matter over a time span Δt (starting at $t=0$).

Gain of thermal energy (l.h.s.) = Joule heating by electric energy (r.h.s.)

$$c_p \cdot M \cdot \Delta T = R \cdot I^2 \cdot \Delta t \quad [\text{Eq. 4}]$$

Because there is no heat loss over the surface, temperature would rise infinitely with time. The unit on both sides is $\text{W}\cdot\text{s}=\text{Joule}$. M and R depend on the geometry of the wire, c_p is a material value.

The electric resistance of a wire is temperature dependent. If the ambient temperature is 20°C , the resistance at any other temperature, T , is typically expressed as

$$R = \rho_{20}(1 + \alpha_{20} \cdot (T - 20)) \frac{L}{A} \quad [\text{Eq. 5}]$$

Using the basic relation for volume and mass we rewrite

$$c_p \cdot \rho \cdot L \cdot A \cdot \Delta T = \rho_{20}(1 + \alpha_{20}(T - 20)) \frac{L}{A} \cdot I^2 \cdot \Delta t \quad [\text{Eq.6}]$$

The length of the wire will cancel. If we assume the initial temperature of the wire also being 20°C (see again note 2), we can unify ΔT and $T-20$ to a temperature rise $\Theta=T-20$. Because of the permanent link between t and Θ , $\Theta(t)$ will be time-dependent and the correct basic equation is an ordinary differential equation:

$$c_p \cdot \rho \cdot \frac{d\Theta}{dt} = \rho_{20}(1 + \alpha_{20}\Theta(t)) \frac{I^2}{A^2} \quad [\text{Eq.7}]$$

Note that I/A is current density (A/m^2).

Solving the Equation:

We solve Equation [7] step by step. First we collect the parameters in an auxiliary quantity K

$$K = \frac{\rho_{20}}{c_p \cdot \rho} \frac{I^2}{A^2}$$

which leads to

$$\frac{d\Theta}{dt} = K(1 + \alpha_{20}\Theta).$$

Then we substitute

$$\mathcal{G} := 1 + \alpha_{20}\Theta \rightarrow \frac{d\Theta}{dt} = \frac{d\Theta}{d\mathcal{G}} \frac{d\mathcal{G}}{dt} = \frac{1}{\alpha_{20}} \frac{d\mathcal{G}}{dt}$$

and get a simple ordinary differential equation,

$$\frac{d\vartheta}{dt} = \alpha_{20} K \vartheta . \quad [\text{Eq. 8}]$$

Initial condition: at time $t=0$ the wire shall be at 20 degC:

$$\Theta(t=0)=0 \rightarrow \vartheta(t=0)=1$$

The way to solve Equation [8] is

$$\int_1^{\vartheta} \frac{d\vartheta'}{\vartheta'} = \int_0^t \alpha_{20} K dt'$$

which gives

$$\ln(\vartheta) - \ln(1) = \alpha_{20} K (t - 0)$$

or (because $\ln(1)=0$)

$$\ln(\vartheta) = \alpha_{20} K t . \quad [\text{Eq. 9}]$$

We then unfold the aux variables

$$\ln(1 + \alpha_{20} \Theta) = \frac{\alpha_{20} \rho_{20}}{c_p \cdot \rho} \frac{I^2}{A^2} \cdot t$$

and exchange rhs and lhs (and move α to the denominator, for reasons that will be apparent in a moment)

$$\frac{\alpha_{20} \rho_{20}}{c_p \cdot \rho} \frac{I^2}{A^2} \cdot t = \ln \left(1 + \frac{\Theta}{1/\alpha_{20}} \right) \quad [\text{Eq. 10}]$$

We are going to solve the l.h.s of this equation separately. First, looking at

$$\frac{\alpha_{20} \rho_{20}}{c_p \cdot \rho} \frac{I^2}{A^2} \quad [\text{Eq. 11}]$$

we rely on the relationship that $\alpha_{T_2} \cdot \rho_{T_2} = \rho_{T_1} \cdot \alpha_{T_1}$ at any two reference temperatures (for a given material.) (See proof in Appendix 3). Therefore, we can use the 20 ° C reference values for ρ and α

regardless of the reference temperature for the equation. Note also that typical expressions for Onderdonk's Equation [3] and [3a] are in units of circular mils. So:

Our material data of copper are:

$$\rho_{20} = 0.0172 \, \Omega \, \text{mm}^2/\text{m} \text{ (at 20 degC) or } 1.75 \cdot 10^{-8} \, \Omega \, \text{m}^2/\text{m}$$

$$\alpha_{20} = 0.00393 \, 1/\text{K}, \text{ (at 20 deg C)}$$

$$c_p = 385 \, \text{J/kgK}$$

$$\rho = 8900 \, \text{kg/m}^3.$$

To calculate the coefficient [Equation 11] in front of $(I/A)^2$ let us use our material data and convert Equation [10] to circular-mils

- a) SI Units : $\frac{\alpha_{20} \cdot \rho_{20}}{c_p \cdot \rho} = 0.00393 \cdot 1.72 \cdot 10^{-8} / (385 \cdot 8900) = 1.973 \cdot 10^{-17}$
- b) $1 \, \text{m}^2 = 1.98 \cdot 10^9 \, \text{circ-mils}$
- c) We have to compensate $A^2 \rightarrow 1 \, \text{m}^4 = 3.9 \cdot 10^{18} \, \text{circ-mils}^2$
- d) Introducing to a) $\rightarrow 1.973 \cdot 10^{-17} \cdot 3.9 \cdot 10^{18} = 76.9$
- e) From $\ln$ to $\log_{10}$ we have to divide by $\ln(10) = 2.30 \rightarrow 76.9 / 2.30 = 33.5$

So, with A in circular mils, is:

$$33.5 \left(\frac{I}{A} \right)^2 \cdot t = \log_{10} \left(1 + \frac{\Theta}{1/\alpha_{20}} \right) \quad [\text{Eq. 12}]$$

Now, to solve for $1/\alpha$ for any reference temperature:

$$\text{We note that } \alpha_{T2} \cdot \rho_{T2} = \rho_{T1} \cdot \alpha_{T1}$$

$$\text{This leads to } 1/\alpha_{T2} = \rho_{T2} / \rho_{T1} \cdot \alpha_{T1} = (\rho_{T1} + \rho_{T1} \cdot \alpha_{T1} (T2 - T1)) / \rho_{T1} \cdot \alpha_{T1}$$

$$\text{Or: } 1/\alpha_{T2} = \frac{\rho_{T1}}{\rho_{T1} \cdot \alpha_{T1}} + (T2 - T1) = \frac{1}{\alpha_{T1}} + (T2 - T1)$$

If we let $T1 = \text{ambient} = 20^\circ \text{C}$ and $T2$ be any other reference temperature, T_{ref} , then

$$1/\alpha_{\text{ref}} = \frac{1}{\alpha_{20}} + T_{\text{ref}} - 20 = 254 - 20 + T_{\text{ref}} = 234 + T_{\text{ref}}$$

Leading to our final expression

$$33.5 \left(\frac{I}{A} \right)^2 \cdot t = \log_{10} \left(\frac{\Theta}{234 + T_{\text{ref}}} + 1 \right) \quad [\text{Eq. 13}]$$

Which at a reference temperature of 40°C (Onderdonk's reference) is

$$33.5 \left(\frac{I}{A} \right)^2 \cdot t = \log_{10} \left(\frac{\Theta}{274} + 1 \right) \quad [\text{Eq. 14}]$$

or equal to Equations [3] and [3a].

It is sometimes desirable to have Equation [13] expressed in terms of t, the time to reach the melting temperature. Solving for t leads to

$$t = \left(\frac{1}{33.5} \right) \left[\log_{10} \left(\frac{\Theta}{234 + T_{ref}} + 1 \right) * \left(\frac{A}{I} \right)^2 \right] \quad [\text{Eq. 15}]$$

where A is still in circular mils.

Using Equation [15], we can now transform Onderdonk's Equation to any temperature reference point (again, see note 2). We offer it with reference temperatures of 20 °C, 40 °C, 85 °C, and 105 °C without further calculations. The latter temperature references are important in the automotive industries. (Note that the higher the reference temperature the shorter the time is to reach the melting temperature.)

Reference Temp °C	Denominator in Log expression
20	254
40	274
85	319
105	339

At the melting point of copper (1083 °C) and at 20 °C, Equation [15] can be reduced to the form:

$$t = c * (A/I)^2 \quad [\text{Eq. 16}]$$

where c = .0213 in circular mils.

The values for c are supplied in in the following table for various units and reference temperatures.

Reference Temp	C in circular mils	C in square mils	C in square mm
20	.0213	.0346	8.30 x 10 ⁻⁴
40	.0203	.0330	7.92
85	.0184	.0298	7.15
105	.0176	.0285	6.85

Notes:

1. A circular mil is the area of a circle with a diameter of one mil. The formula is $A = d^2$. The conversion from circular mils to mil^2 is $\pi/4$. Normal conversions are:
 $1 \text{ mil}^2 = 1.273 \text{ circular mils}$
 $1 \text{ circular mil} = .7854 \text{ mil}^2$
 $1 \text{ m}^2 = 19.736 \times 10^8 \text{ circular mil}$
 $1 \text{ circular mil} = 5.067 \times 10^{-10} \text{ m}^2 = 5.067 \times 10^{-4} \text{ mm}^2$
2. From a thermodynamic engineering standpoint, there is a significant difference between the ambient temperature and the initial condition. For example, there could be some heating of a wire above the ambient temperature in a normal operation before a change of current is applied. In this paper we are assuming that the initial condition (temperature) is the same as the ambient temperature.
3. See http://en.wikipedia.org/wiki/Melting_point
4. l.h.s means left hand side of the equation, r.h.s mean right hand side.

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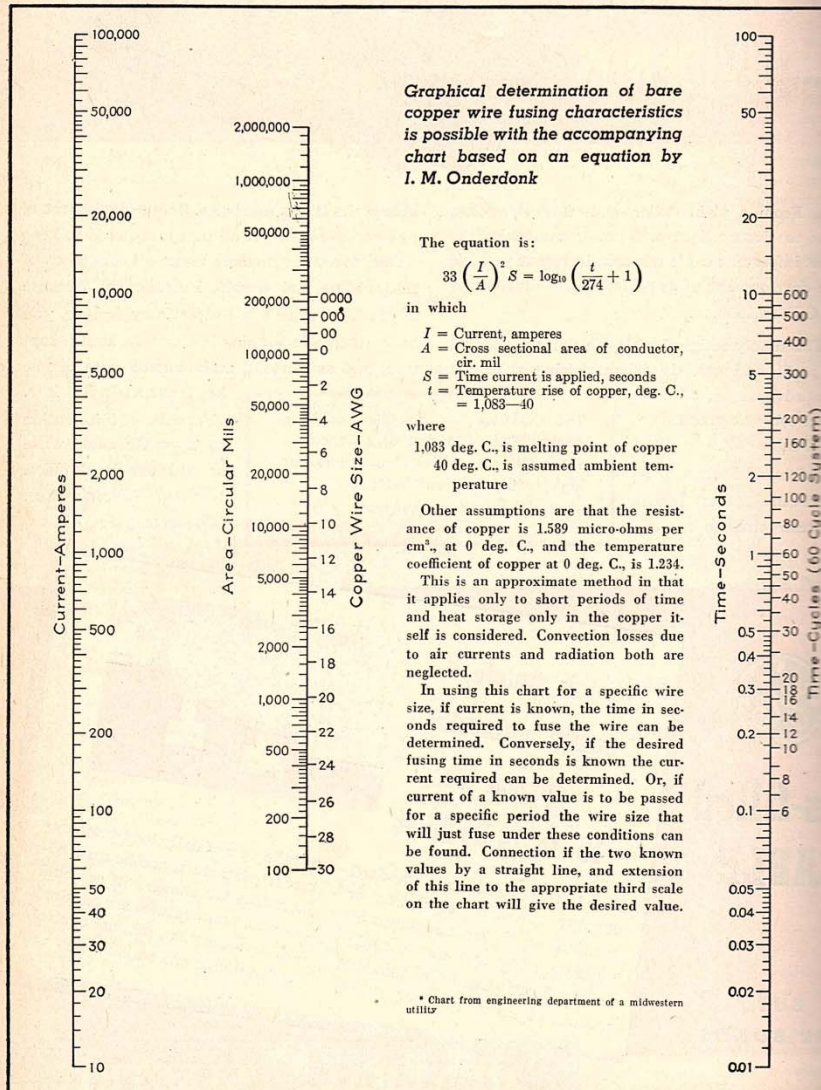
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<http://rspl.royalsocietypublishing.org/content/36/228-231/464.full.pdf+html>
<http://rspl.royalsocietypublishing.org/content/43/258-265/280.full.pdf+html>
<http://rspl.royalsocietypublishing.org/content/44/266-272/109.full.pdf+html>
or at www.ultracad.com/articles/reprints/preece.zip

The following sources (among others) make reference to Onderdonk's equation:

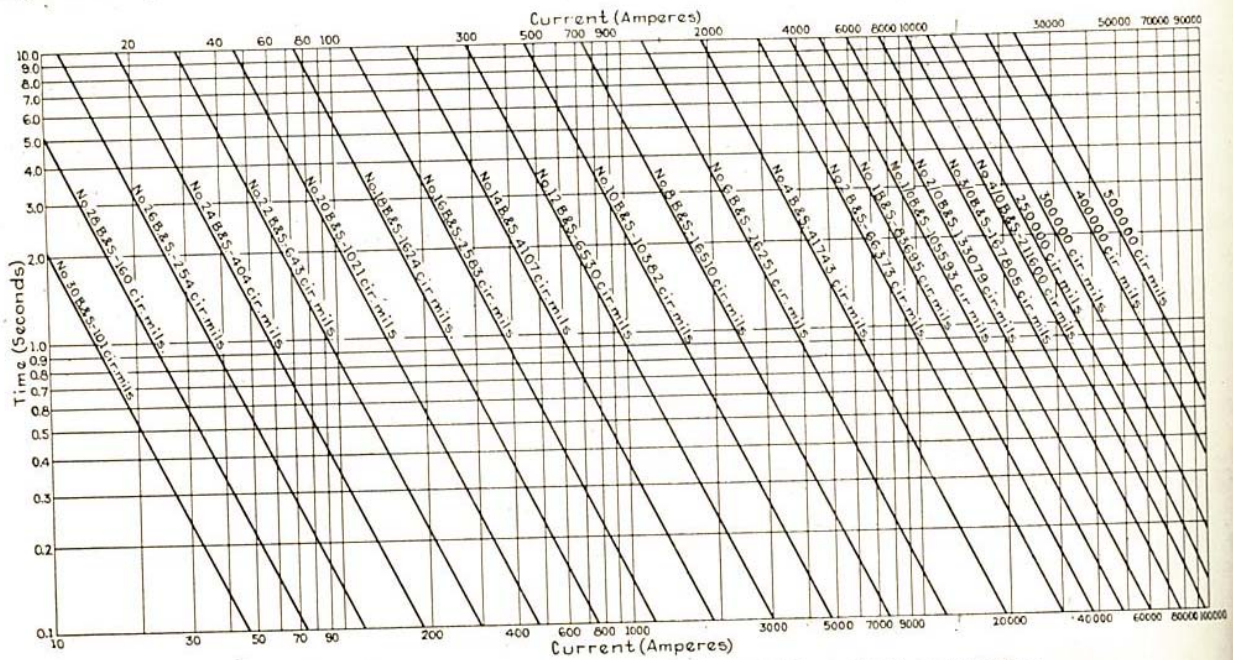
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- [8] Johannes Adam white paper No.10
www.adam-research.de/pdfs/TRM_WhitePaper10_AdiabaticWire.pdf

Appendix 1

SHORT-TIME CURRENT REQUIRED TO MELT COPPER CONDUCTORS*



Appendix 2



Appendix 3

Proof that $\alpha_{T2} * \rho_{T2} = \rho_{T1} * \alpha_{T1}$

Start with the standard relationship for converting resistivity from one reference temperature to another:

$$\frac{\rho_T}{\rho_{Tref}} = 1 + \alpha_{Tref} (T - T_{ref}) \quad \text{or,}$$
$$\rho_T = \rho_{Tref} + \rho_{Tref} * \alpha_{Tref} (T - T_{ref})$$

Let T1, T2, and T3 be any three temperatures. Then the conversion of resistivity between any two temperatures is calculated as follows:

a. T1 to T2: $\rho_{T2} = \rho_{T1} + \rho_{T1} * \alpha_{T1} (T2 - T1)$

b. T1 to T3: $\rho_{T3} = \rho_{T1} + \rho_{T1} * \alpha_{T1} (T3 - T1)$

c. T2 to T3: $\rho_{T3} = \rho_{T2} + \rho_{T2} * \alpha_{T2} (T3 - T2)$

Note that expression b must equal expression c. We can expand expression b as follows:

$$\rho_{T3} = \rho_{T1} + \rho_{T1} * \alpha_{T1} (T3 - T2) + \rho_{T1} * \alpha_{T1} (T2 - T1)$$

Now we set this equation equal to c

$$\rho_{T1} + \rho_{T1} * \alpha_{T1} (T3 - T2) + \rho_{T1} * \alpha_{T1} (T2 - T1) = \rho_{T2} + \rho_{T2} * \alpha_{T2} (T3 - T2)$$

Substitute ρ_{T2}

$$\rho_{T2} + \rho_{T1} * \alpha_{T1} (T3 - T2) = \rho_{T2} + \rho_{T2} * \alpha_{T2} (T3 - T2)$$

Subtract ρ_{T2} and divide each side by (T3-T2) leaves

$$\rho_{T1} * \alpha_{T1} = \rho_{T2} * \alpha_{T2}$$

for any two reference temperatures.

About the authors:



Johannes Adam got a doctorate in physics from University of Heidelberg, Germany, in 1989 on a thesis about numerical treatment of 3- dimensional radiation transport in moving astrophysical plasmas. He was then employed in software companies, mainly working on numerical simulations of electronics cooling at companies like Cisi Ingenierie S.A. , Flomerics. Ltd. and Mentor Graphics Corp. In 2009 he founded ADAM Research and does work as a technical consultant for electronics developing companies and as a software developer. He is the author of a simulation program called TRM (Thermal Risk Management), designed for electronics developers and PCB designers who want to solve electro-thermal problems at the board level. He is a member of the German chapter of IPC (FED e.V.) and engages in its seminars about thermal topics. He is a Certified Interconnect Designer (CID). He lives in Leimen, near Heidelberg.



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